# Boundary-Element Shape Optimization System for Aircraft Structural Components

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## Abstract

An efficient system for the shape optimization of aircraft components based on the boundary-element method is presented. The design sensitivities required by the numerical optimization algorithm are obtained economically by implicit differentiation of the boundary integral equations. The mesh generation and regeneration is done using a parametric and auxiliary geometry concept that allows the original mesh to remain adequate for a wide range of subsequent evolving geometries as the optimization proceeds. The overall system represents an efficient coupling of the geometry definition, mesh generation and regeneration, boundary-element analysis, design sensitivity analysis, geometric consistency maintenance, and numerical optimization. Numerical examples are presented.

## Contents

The boundary-element approach to shape optimization has been surveyed by Kane and Saigal.[1] Most existing shape optimization systems are based on the finite-element method and have been reviewed in Ref. 2. However, no complete system based on the boundary-element method has been reported. An associative shape definition that allows convenient modifications in geometry as the shape evolves to an optimal one and that maintains the geometric integrity of the solid object was developed first. An auxiliary geometry scheme was then implemented that allows the original mesh to be adequate for a wide range of changes in the subsequent geometries. The implicit-differentiation technique[1] was implemented to obtain the design sensitivities. These capabilities were then coupled with the general-purpose numerical optimization code ADS[3] to obtain an overall shape optimization system.

### Associative Shape Definition

In continuum structural shape optimization the shape of the object itself is controlled by the design variables. Therefore, geometric capabilities are an integral part of the overall process and can affect the economy and robustness of a shape optimization system. An associative shape definition was developed to control the shape by a list of design variables that can be continuously modified without destroying the essential properties of that shape. The shape is defined in terms of a control polygon. This polygon is a collection of control points, each of which has been characterized by certain integer-type codes. These codes determine which design variables control the location and radius (if applicable) associated with the control point and the type of geometric entity constructed at the vertex of the control polygon. The default condition is that an edge of the control polygon is also the edge of the part. Exceptions to this rule are specified in two ways:

1) If a finite value of the radius of a control point is encountered, then this control point does not lie on the actual geometry; rather a (circular-arc) filleted junction is constructed to join the two edges of the control polygon at that particular control point.

2) Certain integer-type codes indicate that a group of control points is to define a spline curve. If desired, splines can be made tangent to adjacent curves. Because the polygon points are controlled by a design variable list, the numerical optimization procedure can affect a change in the shape of the design while preserving the essential geometric consistency of the design by simply altering this list. This is because the intrinsic geometric characteristics of a shape (for example, circular arc tangent to two straight lines) are associated with the control polygon rather than occurring coincidentally. Techniques for guarding against geometric inconsistencies are an indispensable prerequisite for any geometry module used for a shape optimization system. The geometric consistencies are included as inequality constraints in the numerical optimization program.

### Auxiliary Geometry, Meshing and Remeshing

For a two-dimensional shape, meshing and remeshing must be done only for the curves that bound the two-dimensional area and the lines forming the interzone boundaries. A versatile boundary-element mesh generation system for shape optimization should include a capability that allows the mesh to remain valid for a wide range of shape variations. A technique that involves three additional geometric entities for the shape definition to allow for effective meshing and remeshing during the shape optimization process was developed. These three geometric entities, referred to collectively as auxiliary geometry, are: auxiliary points, auxiliary lines, and composite curves. Auxiliary points are used to connect auxiliary lines or to serve as markers for the endpoints of composite curves. Auxiliary lines are used to form interzone boundaries in a multizone boundary-element model. Composite curves are a contiguous collection of segments of lines, arcs, or splines considered as one geometric entity. When such auxiliary quantities are included in the shape definition, the range of shape variation for which the original boundary element remains adequate is extended significantly. The auxiliary geometry definition described earlier then allows efficient automatic remeshing without user intervention as new geometric shapes are generated during shape optimization.

### Design Sensitivity Analysis

Discretizing Somigliana's identity, which describes a boundary integral relationship governing the elastic response of a solid object, and then performing the implicit differentiation of this discretized equation leads to the relationship

$$[H]\{U\}_{,L} = [G]\{T\}_{,L} + [G]_{,L}\{T\} - [\mathrm{H}]_{,L}\{U\}$$

where $[H]$ and $[G]$ are the boundary-element system matrices, $\{T\}$ are the vectors of boundary tractions and displacements, respectively, and the subscript $L$ denotes differentiation with

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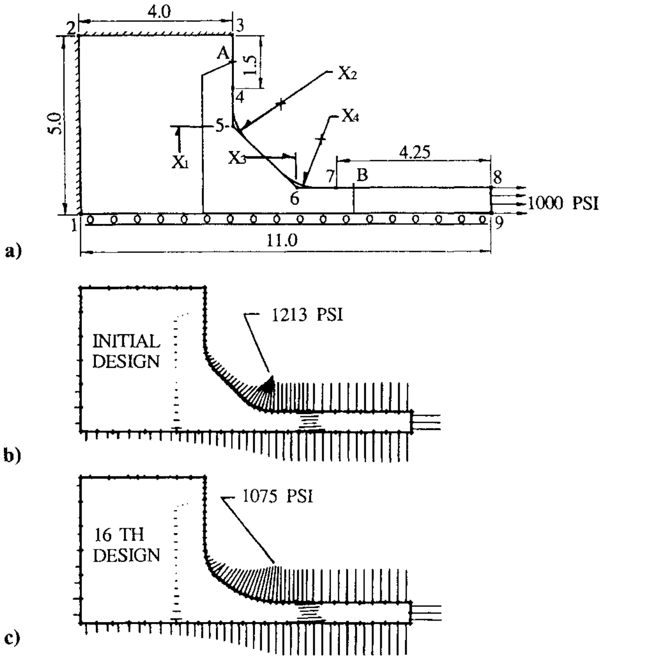


**Fig. 1 Transition region of a rod to half-space attachment; dimensions in inches.**

respect to the design variable $X_L$. The solution of Eq. (1) provides the design sensitivity information. Further details regarding determination of system matrices in Eq. (1) and the solution algorithm are described fully in Ref. 1.

### Illustrative Examples

The structural analysis and the sensitivity analysis modules of the present system were coupled to the numerical optimization algorithm ADS[3]. The present system was used for the shape optimization of a set of example cases.

#### *Transition Region of a Rod to Half-Space Attachment*

A symmetric half of a rod that is attached to a wall and subjected to an axial pull of 1000 psi is shown in Fig. 1a. The transition area where the road meets the wall exhibits stress concentration; this was the region whose shape was of concern. The four design variables that were used to define this transition region were shown in Fig. 1a. The object was modeled using three boundary-element zones (58 boundary elements, 147 nodal points). The volume of the material in the attachment zone was chosen to be the objective function in this example. Geometric consistency constraints along with limits on Von Mises stress of 1100 psi at the sample points along the curved region AB were also imposed. The Von Mises stress distribution for the initial design is shown in Fig. 1b. In this and subsequent figures, the stress at any location is depicted as a line drawn normal to the boundary, with its length proportional to the stress value at that point. The initial design was infeasible since the maximum stress of 1213 psi for that design was above the upper prescribed limit. An improving sequence of designs was generated and examined by the shape optimization system. The final shape given by the system is shown in Fig. 1c.

#### *Slotted Ring in Aircraft Gas Turbine Engine*

A thin slotted ring with many cooling slots as shown in Fig. 2a was analyzed next. Because the ratio of radial thickness to average radius for the ring is small, it was idealized as a flat strip in a state of plane stress. An axial displacement was applied to produce a stress of 10 ksi along the center of the ring. The slotted ring has a repeating geometry and, therefore, only a symmetric quarter of the repeated cell was modeled using a three-zone boundary-element mesh. As the shape of the curved region in this geometry evolves, any portion along the boundary could become a critical location of high curvature or large variation in stress or both. A uniformly fine mesh was therefore provided for this region because such critical regions are not known a priori. A geometric constraint, requiring tangency of the segments where they meet the straight lines, was imposed. Limits on values of the design variables were specified, and geometric consistency constraints were also imposed. A smooth stress distribution along the curved boundary was chosen as the objective function. The sum of the powers of tangential stress at all nodes along the curved boundary was used to quantify this objective function. A control polygon with five design variables shown in Fig. 2b was first used. The initial design has a circular fillet radius slightly smaller than the slot half-width. The original design shown in Fig. 2c is in the unconstrained portion of the design space and is thus feasible. The shape optimization system was employed to improve this initial design. A sequence of shapes was generated and examined by the system. A desired reduction in stress value with evolving shapes was observed, and the final configuration after five steps was shown in Fig. 2d. This example points out the type of designs that can occur when optimization systems are given problems with too much freedom. The "bump" shown in Fig. 2d can be forced not to develop by simply including additional appropriate geometric side constraints in this region. The same component was again optimized with a different control polygon using a new set of design variables. A reduction in the value of objective function from 23.71 to 16.91 ksi in four iterations was obtained. This model, which had fewer design variables, produced a better final design compared to the previous model, which had more design variables.

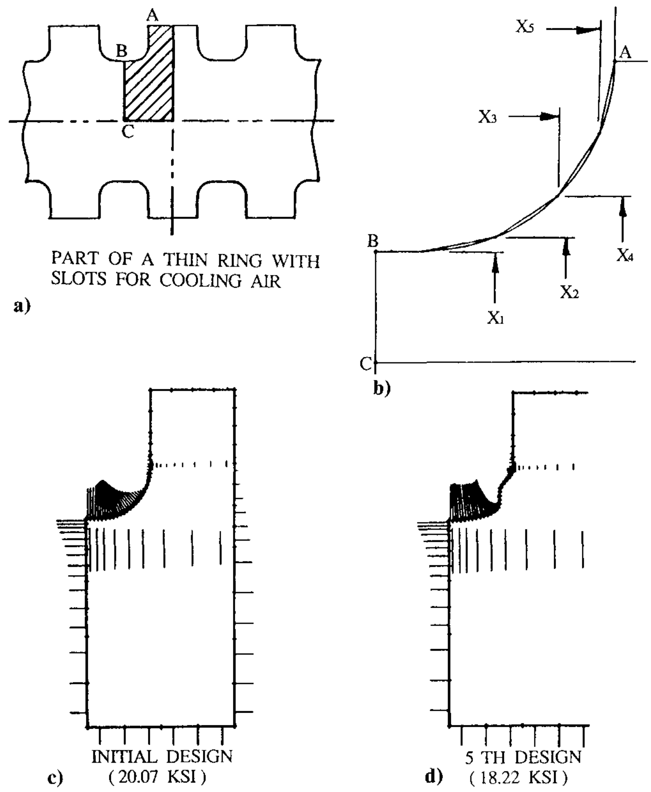


**Fig. 2 Slotted ring in aircraft gas turbine engine.**

## References


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